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the lectures pdfs are available at:

https://www.physics.umd.edu/rgroups/amo/orozco/results/2022/Results22.htm

Correlations in Optics and Quantum Optics; A series of lectures about correlations and coherence 5. November 2022

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BOS.QT

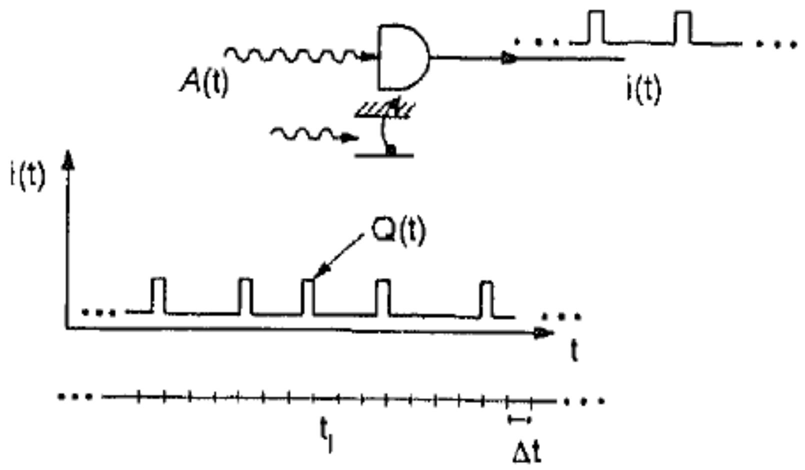
Lesson 5

Tentative list of topics to cover:

- From statistics and linear algebra to power spectral densities
- Historical perspectives and examples in many areas of physics
- Correlation functions in classical optics (field-field; intensity-intensity; field-intensity) part iii
- Optical Cavity QED
- Correlation functions in quantum examples
- Correlations of the field and intensity
- Correlations and conditional dynamics for control
- From Cavity QED to waveguide QED.

More about shot noise

Light detection (shot noise)



H. J. Kimble
Les Houches
1990

The field A(t) produces a photocurrent with charge Q. Ionization happens in a period Δt such that it is small enough to only have one electron in each Δt. p_k is a random variable

$$i(t) = \sum_k Q(t - t_k) p_k .$$

We have to calculate the power spectral density

$$\Phi(\Omega) \equiv \int \langle \Delta i(t) \Delta i(t+\tau) \rangle \, e^{-i\Omega\tau} \, d\tau,$$

Correlation function

$$\langle \Delta i(t) \Delta i(t+\tau) \rangle = \langle i(t) i(t+\tau) \rangle - \langle i \rangle^2$$

The correlation is:

$$\langle i(t) i(t+\tau) \rangle = \left\langle \sum_{k=-\infty}^{\infty} Q(t-t_k) p_k \sum_{j=-\infty}^{\infty} Q(t+\tau-t_j) p_j \right\rangle$$

H. J. Kimble

$$= \sum_k Q(t - t_k)Q(t + \tau - t_k)\langle p_k \rangle + \sum_{k \neq j} Q(t - t_k)Q(t + \tau - t_j)\langle p_k p_j \rangle,$$

$$\langle p_1 p_2 \dots p_k \rangle = W_k(t_1, t_2, \dots, t_k)\Delta t_1 \Delta t_2 \dots \Delta t_k,$$

$$W_k(t_1, t_2, \dots, t_k) = \alpha^k \langle : I(t_1)I(t_2) \dots I(t_k) : \rangle$$

$$Q(t - t') = Q_0 \delta(t - t'),$$

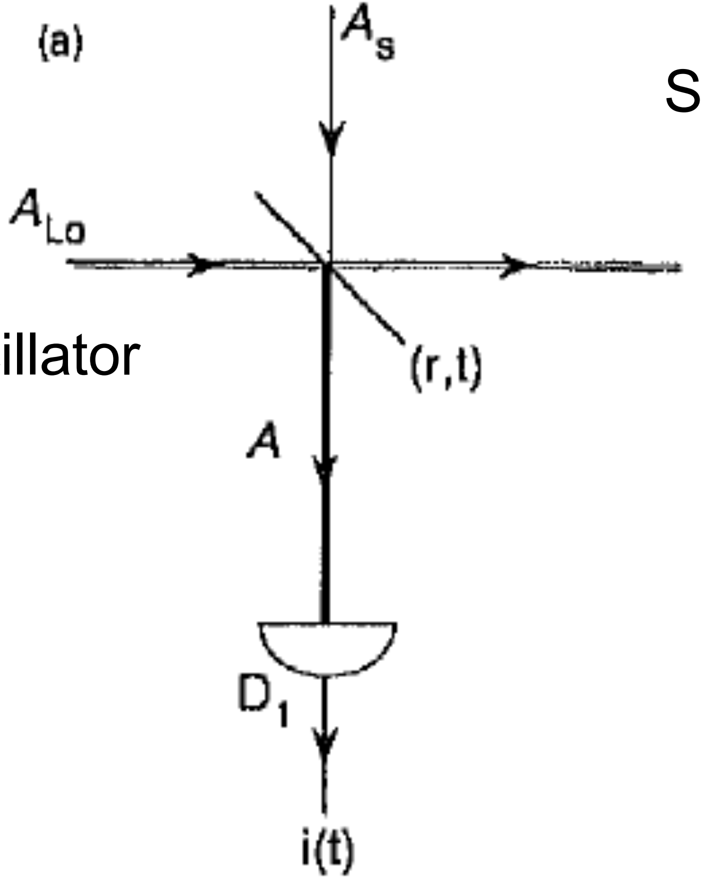
What Glauber taught us

$$
\begin{aligned}
\langle i(t)i(t+\tau)\rangle = & \int \mathrm{d}t'\ W_1(t')Q(t-t')Q(t+\tau-t') \\
& + \int \mathrm{d}t' \int \mathrm{d}t''\ W_2(t',t'')Q(t-t')Q(t+\tau-t'') \\
= & Q_0^2\alpha\ \langle : I(\tau) : \rangle \delta(\tau) \\
& + Q_0^2\alpha^2\ \langle : I(t)I(t+\tau) : \rangle .
\end{aligned}
$$

$$
\langle \Delta i(t)\Delta i(t+\tau)\rangle = Q_0^2\alpha\ \langle : I(t) : \rangle\ \delta(\tau) + Q_0^2\alpha^2 C(\tau)
$$

$$
C(\tau) \equiv \langle : I(t)I(t+\tau) : \rangle - \langle : I : \rangle^2 .
$$

The photocurrent could be due to beating, arbitrary local oscillator power

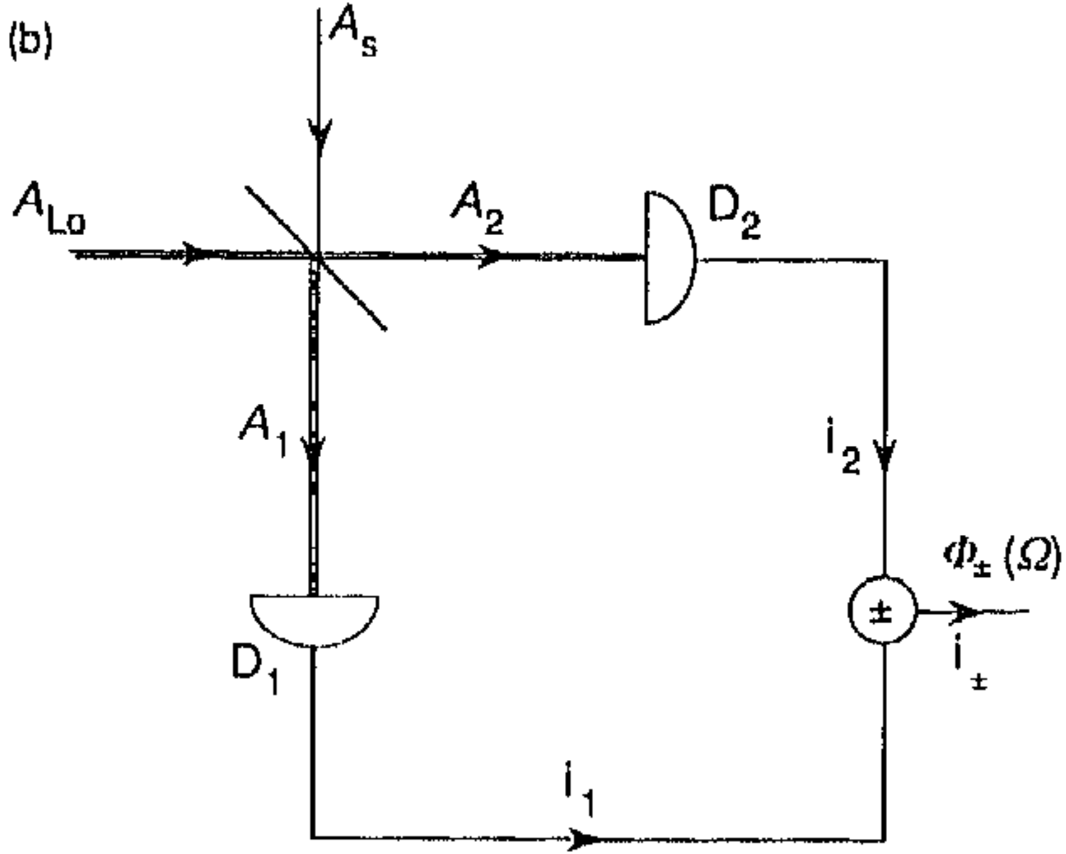


Signal

Local Oscillator

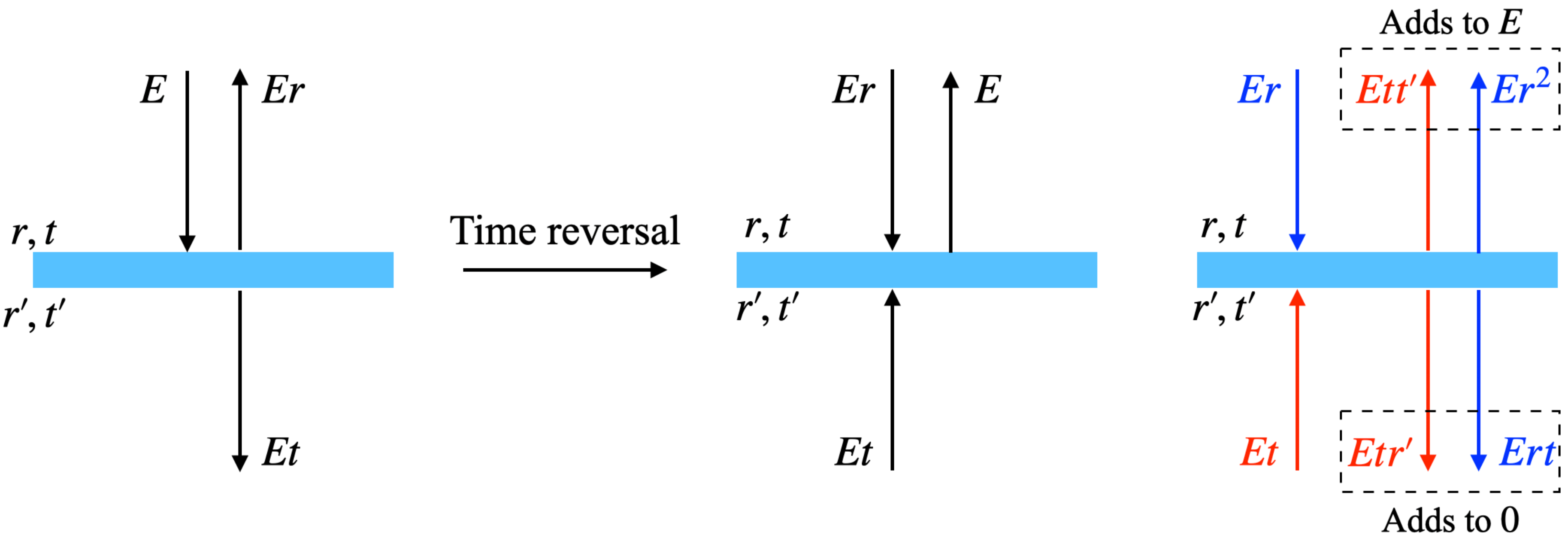
t>>r
$R=r^2$
$T=t^2$

The balanced homodyne detector



common noise of the LO eliminated

Stokes argument



So $tt' + r^2 = 1$, $r + r' = 0$

Fresnel relations: $$r = \frac{n_2 - n_1}{n_2 + n_1}$$

the correlation with the A field

$$A = rA_{\mathrm{LO}} + tA_{\mathrm{s}}.$$

First order in A_s

$$C(\tau) = RTA_0^2[e^{-2i\theta} \langle A_{\mathrm{s}}(t+\tau), A_{\mathrm{s}}(t)\rangle + e^{2i\theta}\langle A_{\mathrm{s}}^\dagger(t), A_{\mathrm{s}}^\dagger(t+\tau)\rangle + \langle A_{\mathrm{s}}^\dagger(t), A_{\mathrm{s}}(t+\tau)\rangle + \langle A_{\mathrm{s}}^\dagger(t+\tau), A_{\mathrm{s}}(t)\rangle],$$

But the fluctuations of the quadratures of the electromagnetic field:

$$z_\theta(t) = e^{-i\theta} A_{\mathrm{s}}(t) + A_{\mathrm{s}}^\dagger(t)\, e^{i\theta},$$

$$C(\tau) = RTA_0^2 \langle : z_\theta(t), z_\theta(t+\tau) : \rangle$$

$$\langle \Delta i(t) \Delta i(t+\tau) \rangle = Q_0 i_0 [\delta(\tau) + \alpha T \langle : z_\theta(t), z_\theta(t+\tau) : \rangle],$$

$$\Phi(\Omega, \theta) = Q_0 i_0 [1 + \alpha T S_\mathrm{s}(\Omega, \theta)],$$

$$S_\mathrm{s}(\Omega, \theta) = \int d\tau \, \mathrm{e}^{-\mathrm{i}\Omega\tau} \langle : z_\theta(t), z_\theta(t+\tau) : \rangle.$$

This is the spectrum of squeezing

The power spectral density of shot noise when S=0:

$$\langle(\Delta i(\Omega,\theta))^2\rangle = \frac{1}{2\pi}\left[\int_{-\Omega-\Delta\Omega/2}^{-\Omega+\Delta\Omega/2} \Phi(\Omega,\theta)\,\mathrm{d}\Omega + \int_{\Omega-\Delta\Omega/2}^{\Omega+\Delta\Omega/2} \Phi(\Omega,\theta)\,\mathrm{d}\Omega\right]$$

$$\langle(\Delta i(\Omega))^2\rangle = 2Q_0 i_0 B.$$ B=Bandwidth

But if there is squeezing S≠0

$$\langle(\Delta i(\Omega))^2\rangle = 2Q_0 i_0 B[1 + \xi S(\Omega,\theta)]$$

Taking into account propagation, α detection, η beating efficiency, and ρ cavity exit efficiency, transmission from generator to detectors T_0

$$\xi \equiv \alpha\eta^2 T_0 \rho$$

From a historical perspective the noise equations are reminiscent of the 1909 Einstein paper on black body radiation analyzing the energy fluctuations for a black body. Einstein found that the variance in energy $\langle(\Delta E)^2\rangle$within some small volume and frequency interval could be written in terms of particle plus wave contributions as:

$$\langle(\Delta E)^2\rangle = (\hbar\omega)^2\langle m\rangle + \left\langle(\Delta I)^2\right\rangle$$

First term are the particle fluctuations and the second the wave fluctuations

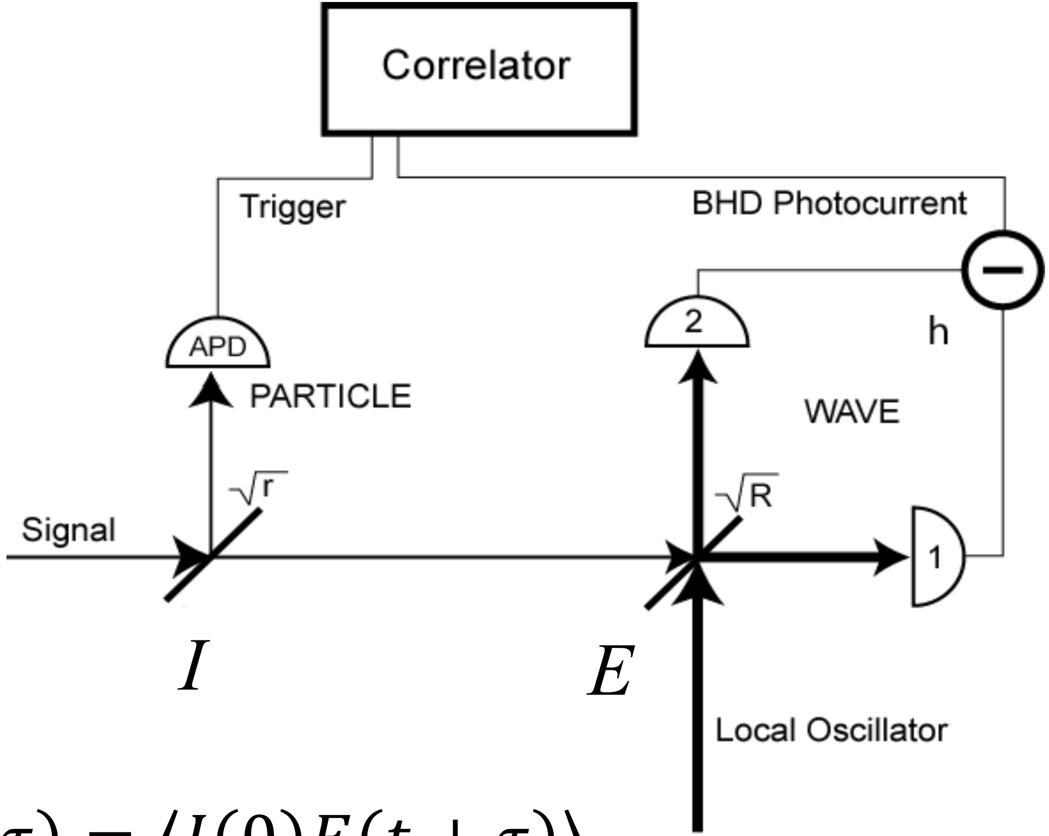
$$\langle\Delta i(t)\Delta i(t+\tau)\rangle = Q_0 i_0[\delta(\tau) + \alpha T\langle: z_0(t), z_0(t+\tau) :\rangle],$$

How to correlate fields and intensities?

Detection of the field: Homodyne.

Conditional Measurement: Only measure when there is a large flucuation.

The Intensity-Field correlator.



$$H(\tau) = g^{3/2}(\tau) = \langle I(0)E(t+\tau)\rangle$$

Balanced homodyne detector suppresses technical common noise.

The correlator is a digital storage oscilloscope;
Only average the photocurrent if there is a fluctuation.

The average of noise is zero.

$$G^{(1)}(\tau) := \langle \mathcal{E}^*(t)\mathcal{E}(t+\tau)\rangle \tag{4}$$

$$G^{(3/2)}(\tau) := \langle \mathcal{E}^*(t)\mathcal{E}(t)\mathcal{E}^*(t+\tau)\mathcal{E}_{lo}(t+\tau)\rangle + c.c. \tag{5}$$

$$G^{(2)}(\tau) := \langle \mathcal{E}^*(t)\mathcal{E}(t)\mathcal{E}^*(t+\tau)\mathcal{E}(t+\tau)\rangle. \tag{6}$$

Where $\mathcal{E}_{lo}(t) = A_{lo}\exp(-i\omega_{lo}t)$, with $A_{lo} = E_{lo}\exp(i\theta)$, is a coherent local oscillator with the same deterministic mode $\omega_{lo} = \langle\omega\rangle$ than the field.

$$g^{(1)}(\tau) := \frac{\langle \mathcal{E}^*(t)\mathcal{E}(t+\tau)\rangle}{\langle |\mathcal{E}(t)|^2\rangle}$$

$$g^{(3/2)}(\tau) := \frac{1}{2}\frac{\langle \mathcal{E}^*(t)\mathcal{E}(t)[\mathcal{E}^*(t+\tau)\mathcal{E}_{lo}(t+\tau) + \mathcal{E}(t+\tau)\mathcal{E}_{lo}^*(t+\tau)]\rangle}{\langle \mathcal{E}^*(t)\mathcal{E}(t)\rangle \,|\langle A(t)\rangle|\,|\langle A_{lo}(t)\rangle|},$$

$$g^{(2)}(\tau) := \frac{\langle \mathcal{E}^*(t)\mathcal{E}(t)\mathcal{E}^*(t+\tau)\mathcal{E}(t+\tau)\rangle}{\langle \mathcal{E}^*(t)\mathcal{E}(t)\rangle \langle \mathcal{E}^*(t+\tau)\mathcal{E}(t+\tau)\rangle}.$$

Let $E_\beta \exp(i\phi)$ be the non-zero average steady part in the complex amplitude of a field $\mathcal{E}(t)$. Let $\delta A(t)$ be the fluctuations of the complex amplitude, we assume that the moments of third order for the fluctuations $\{\delta A(t)\}$ are negligible compared to the lower orders. The intensity-field correlation function $g^{(3/2)}(\tau)$ is classically defined by (8). Because of the steady part $E_\beta \exp(i\phi)$ in the complex amplitude of the field, both the numerator and denominator in (8) are non zero. We will show that this correlation function captures the evolution of a quadrature of the field, depending on the relative phase $(\phi - \theta)$ between the local oscillator and the field.

$$A_\mu(t) := \frac{1}{\sqrt{2}}[A(t)\exp(-i\mu) + A^*(t)\exp(i\mu)].$$

The capture of a quadrature evolution is conditioned on an intensity fluctuation because of the term $\mathcal{E}(t)\mathcal{E}^*(t)$ and its c.c.

$$G^{(3/2)}(\tau) := \langle \mathcal{E}^*(t)\mathcal{E}(t)\mathcal{E}^*(t+\tau)\mathcal{E}_{lo}(t+\tau)\rangle + c.c.$$

$$g^{(3/2)}(\tau) \approx \cos(\phi - \theta) + \frac{\langle \delta A_\phi(t) \delta A_\theta(t+\tau) \rangle}{E_\beta^2 + \langle |\delta A(t)|^2 \rangle}.$$

When the phases are equal ($\phi = \theta$) it becomes:

$$g^{(3/2)}(\tau) \approx 1 + \frac{\langle \delta A_\theta(t) \delta A_\theta(t+\tau) \rangle}{E_\beta^2 + \langle |\delta A(t)|^2 \rangle}.$$

These are the fluctuations of the quadrature

The calculation of the S/N ratio requires taking into consideration what is the signal. Usually the difference between $g^{(n)}(0)$ and $g^{(n)}(t>>0)$.

Also the response function of the detector

Shot and Technical noise.

The classical correlations are related one to the other if the random process is Gaussian or similar.

$$\Gamma^{(2)}(\tau) \equiv \langle I(t)I(t+\tau)\rangle = \langle E^*(t)E^*(t+\tau)E(t+\tau)E(t)\rangle.$$

times. For a thermal source with Gaussian statistical distribution, we can apply the Gaussian moment theorem (see Isserlis theorem in Eq. (1.28)):

$$\langle ABCD\rangle = \langle AB\rangle\langle CD\rangle + \langle AC\rangle\langle BD\rangle + \langle AD\rangle\langle BC\rangle. \tag{4.27}$$

$$\begin{aligned}\Gamma^{(2)}(\tau) = &\langle E^*(t)E^*(t+\tau)\rangle\langle E(t+\tau)E(t)\rangle \\ &+\langle E^*(t)E(t+\tau)\rangle\langle E^*(t+\tau)E(t)\rangle \\ &+\langle E^*(t)E(t)\rangle\langle E^*(t+\tau)E(t+\tau)\rangle \\ = &|\Gamma(\tau)|^2 + |\Gamma(0)|^2,\end{aligned}$$

where, due to the randomness of the phase of the field, $\langle E^*(t)E^*(t+\tau)\rangle = 0 = \langle E(t+\tau)E(t)\rangle$. Hence, the normalized intensity correlation function is

$$g^{(2)}(\tau) \equiv \frac{\langle E^*(t)E^*(t+\tau)E(t+\tau)E(t)\rangle}{\langle E^*(t)E(t)\rangle\langle E^*(t+\tau)E(t+\tau)\rangle}$$
$$= \Gamma^{(2)}(\tau)/|\Gamma(0)|^2 = 1 + |\gamma(\tau)|^2, \qquad (4.29)$$

Condition on a Click

Measure the correlation function of the Intensity and the Field:

$\langle I(t)\, E(t+\tau)\rangle$

Normalized form:

$h_\theta(\tau) = \langle E(\tau)\rangle_c / \langle E\rangle$

From Cauchy Schwartz inequalities:

$$0 \leq \bar{h}_0(0) - 1 \leq 2$$

$$\left|\bar{h}_0(\tau) - 1\right| \leq \left|\bar{h}_0(0) - 1\right|$$

Question:

Is the S/N ratio better in $g^{(3/2)}$ than in $g^{(2)}$ for measurements of the diameter of stars (classical sources)?

A. Siciak, M. Hugbart, W. Guerin, R. Kaiser, L. A. Orozco "A comparison of $g^{(1)}(\tau)$, $g^{(3/2)}(\tau)$, and $g^{(2)}(\tau)$ for radiation from harmonic oscillators in Brownian motion with a coherent background, " Physica Scripta **95**, 104001 (2020) https://doi.org/10.1088/1402-4896/abac37 pdf

Thanks

Correlation functions tell us something about the fluctuations.

Correlations have classical bounds.

They are conditional measurements.

Can we use them to measure the field associated with a FLUCTUATION of one photon?

Correlation function; Conditional measurement.

Detect a photon: Now follow the evolution of the conditional quantum mechanical state in the system.

The system has to have at least two photons.

Do we have enough signal to noise ratio?

$|LO|^2$ + $2\ LO\ S \cos(\phi)$

SHOT NOISE SIGNAL

Relation with the harmonic oscillaros:

Quadratures of the electromagnetic field

$$E_{\mathrm{R}} = \left(\frac{\hbar\omega}{2\epsilon_0 V}\right)^{1/2} \cos(\theta) X \qquad \text{and} \qquad E_{\mathrm{I}} = \left(\frac{\hbar\omega}{2\epsilon_0 V}\right)^{1/2} \sin(\theta) X$$

$$H = \hbar\omega \left(P^2 + X^2\right), \qquad \text{with} \qquad [X, P] \equiv XP - PX = \frac{i}{2} I$$

$$\left(X - \langle X\rangle\right) |\alpha\rangle = -i \left(P - \langle P\rangle\right) |\alpha\rangle$$

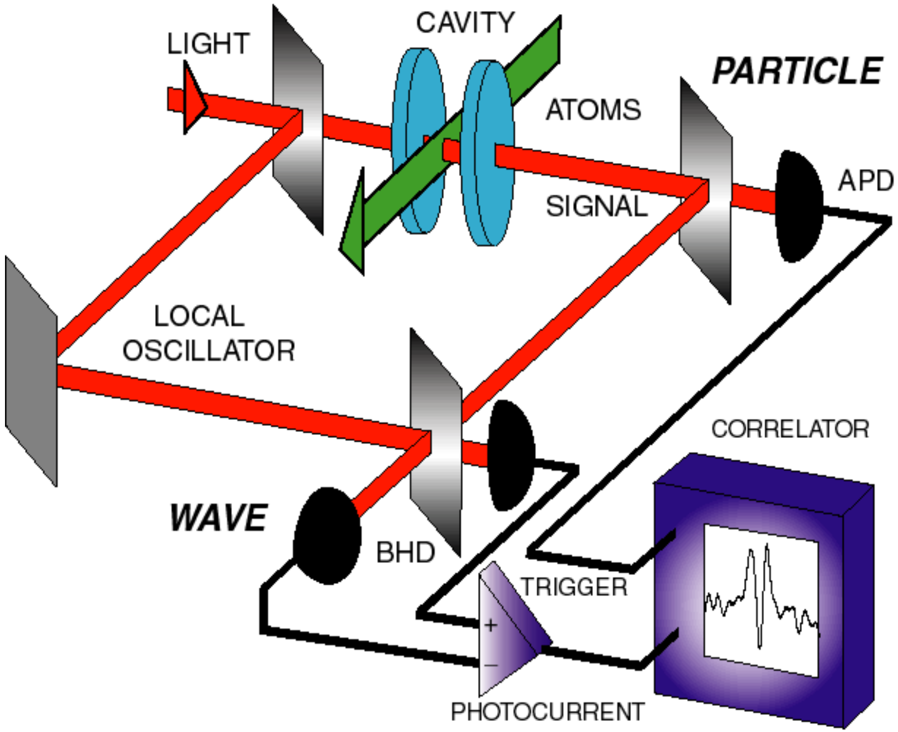
$$\left(X + iP\right) |\alpha\rangle = \langle X + iP\rangle \, |\alpha\rangle$$

States of minimum uncertainty :

$$\langle\alpha| \left(X - \langle X\rangle\right)^2 + \left(P - \langle P\rangle\right)^2 |\alpha\rangle = 1/2$$

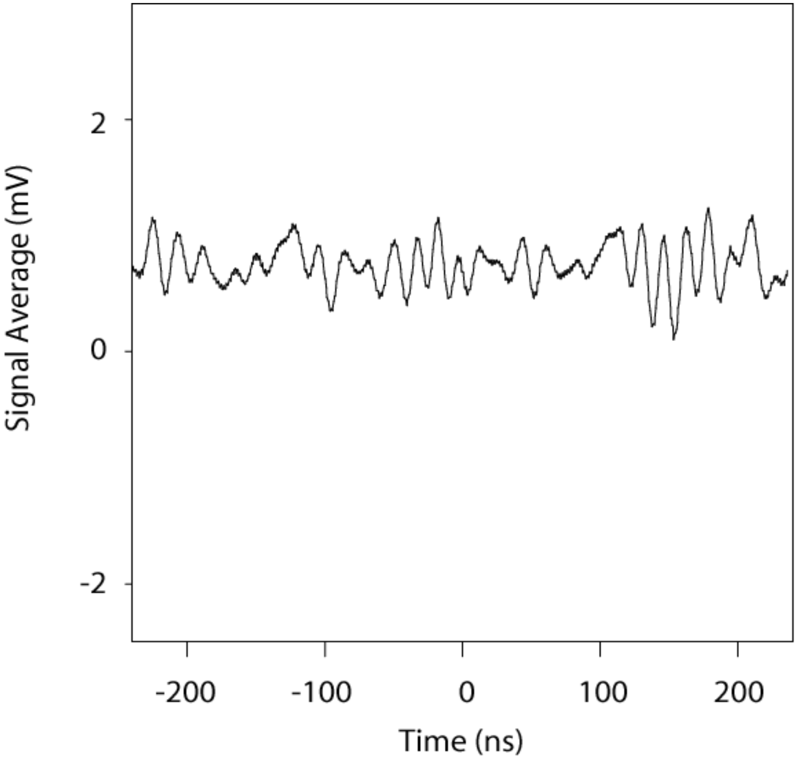
Relation with Fock states (Poisson)

$$P(n) = |\langle n|\alpha\rangle|^2 = e^{-\langle n\rangle} \frac{\langle n\rangle^n}{n!}$$

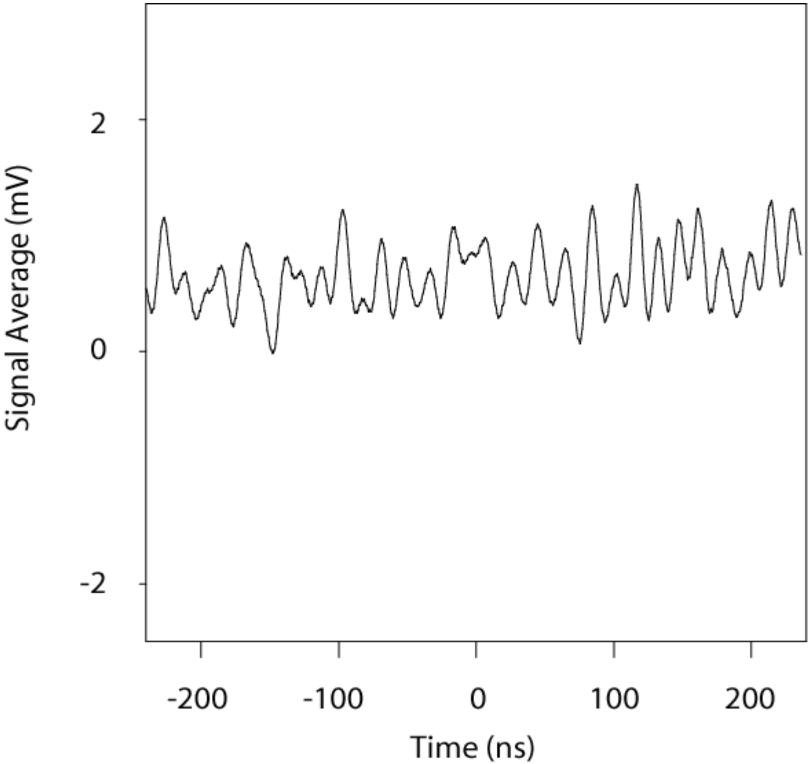


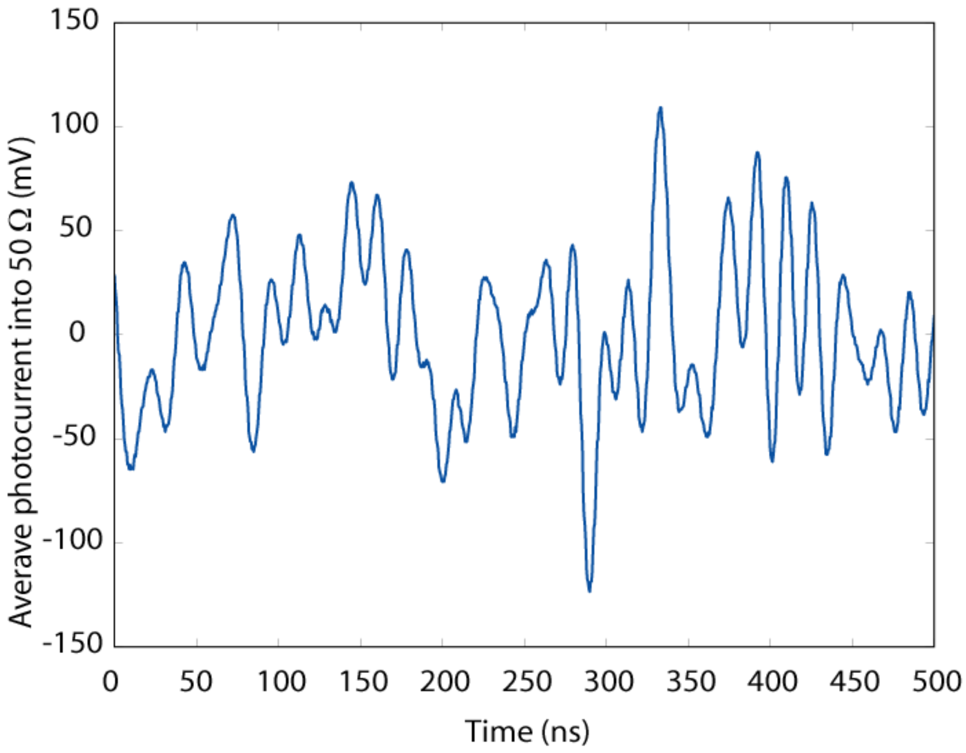
CAVITY
LIGHT
PARTICLE
ATOMS
APD
SIGNAL
LOCAL
OSCILLATOR
CORRELATOR
WAVE
BHD
TRIGGER
+
–
PHOTOCURRENT

Photocurrent average with random conditioning

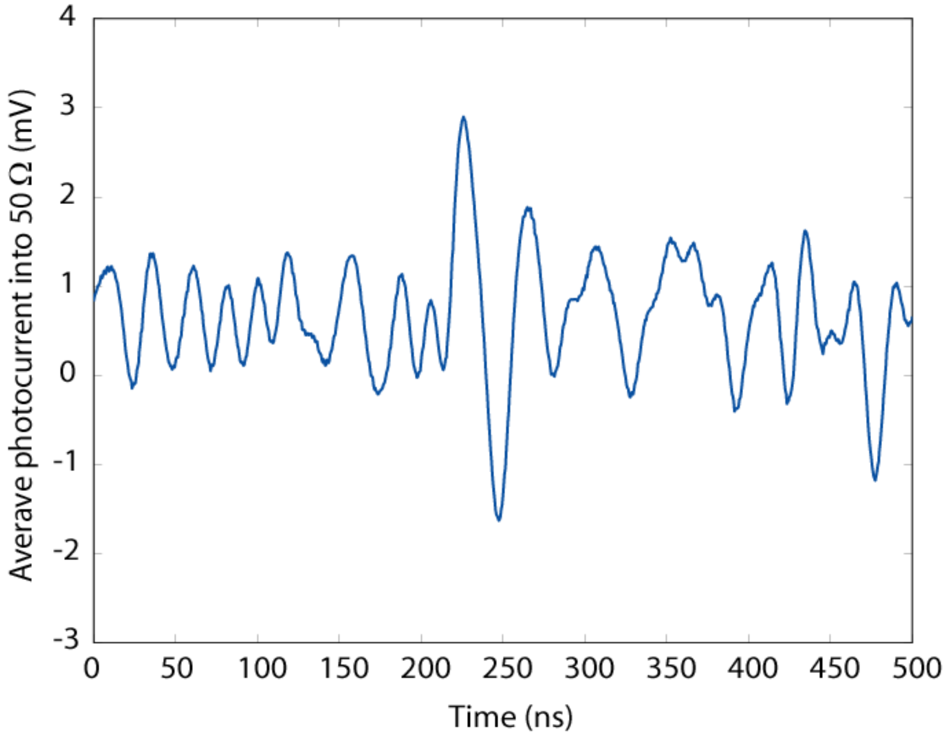


Conditional photocurrent with no atoms in the cavity.

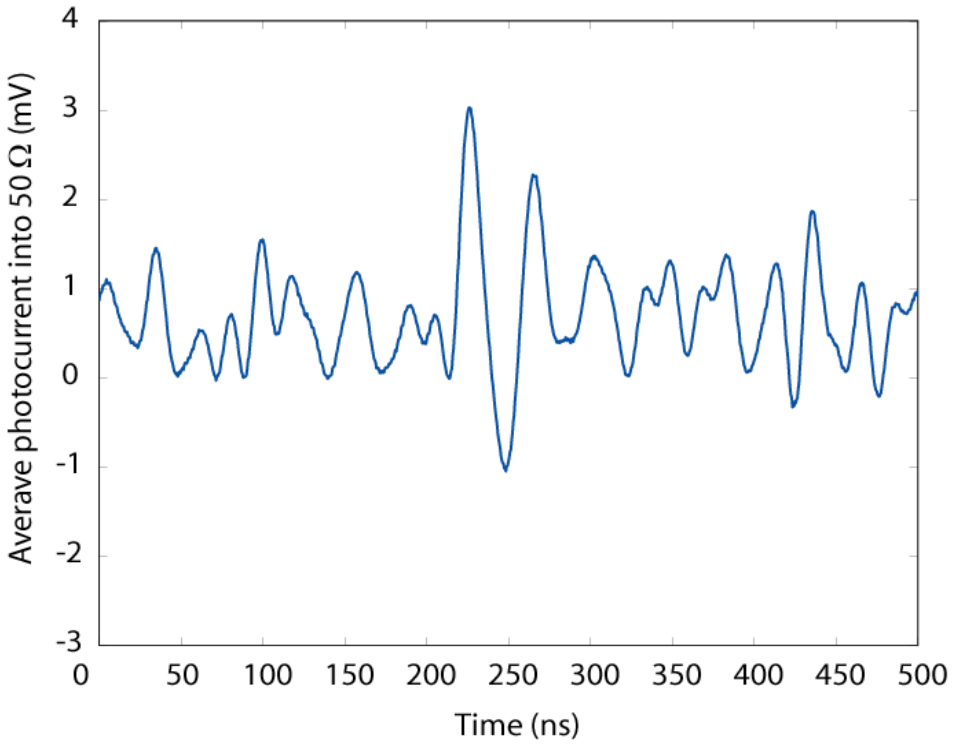




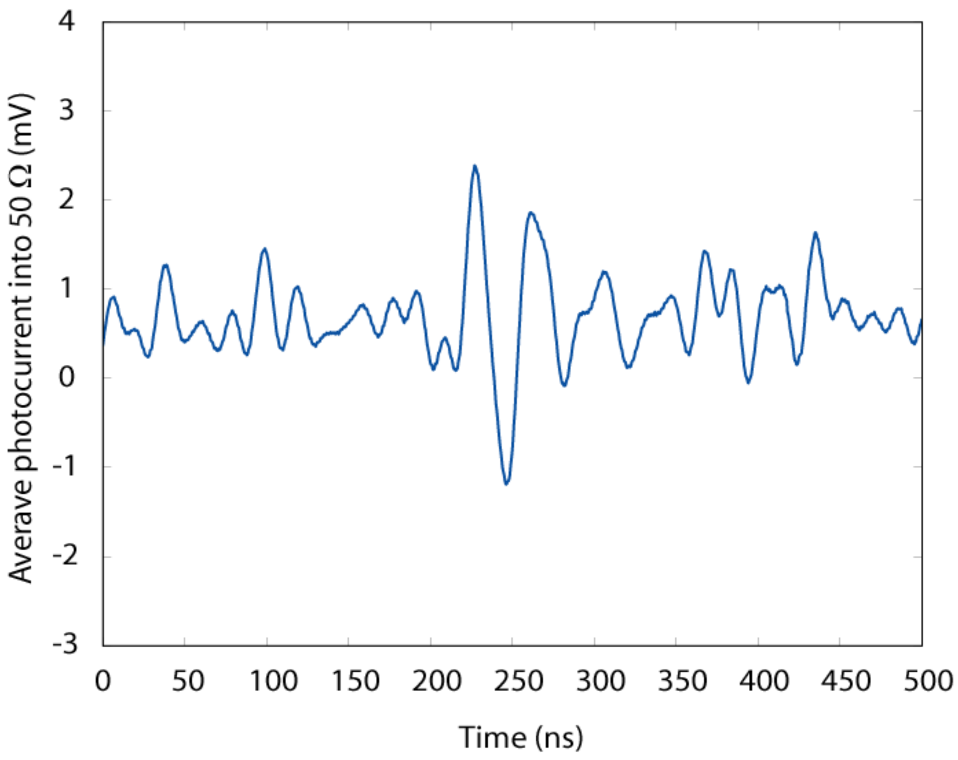
After 1 average



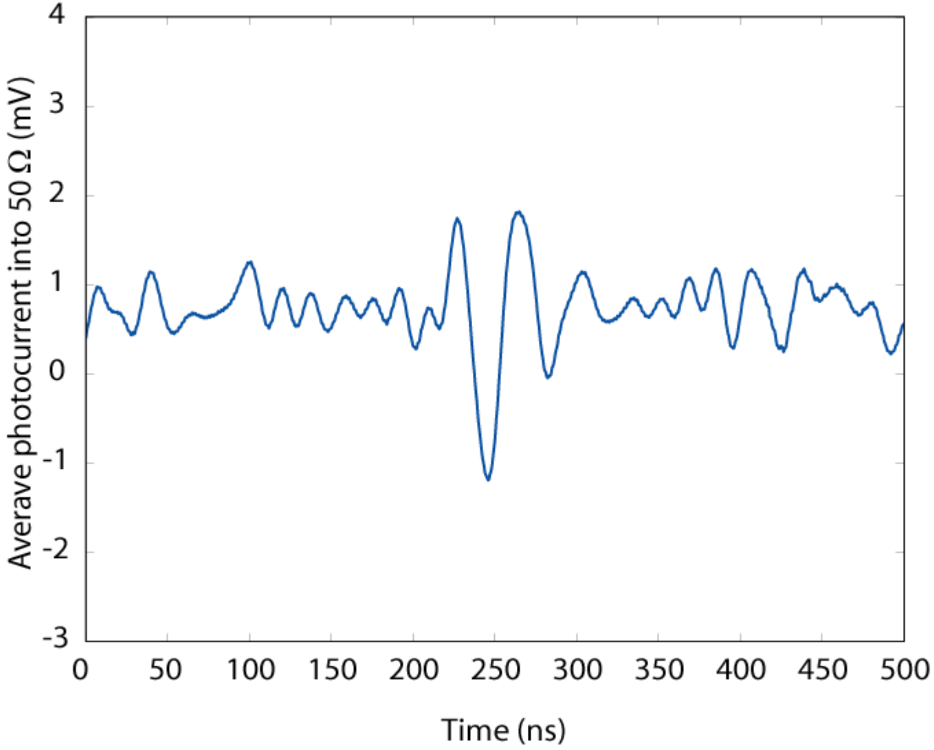
After 6,000 averages



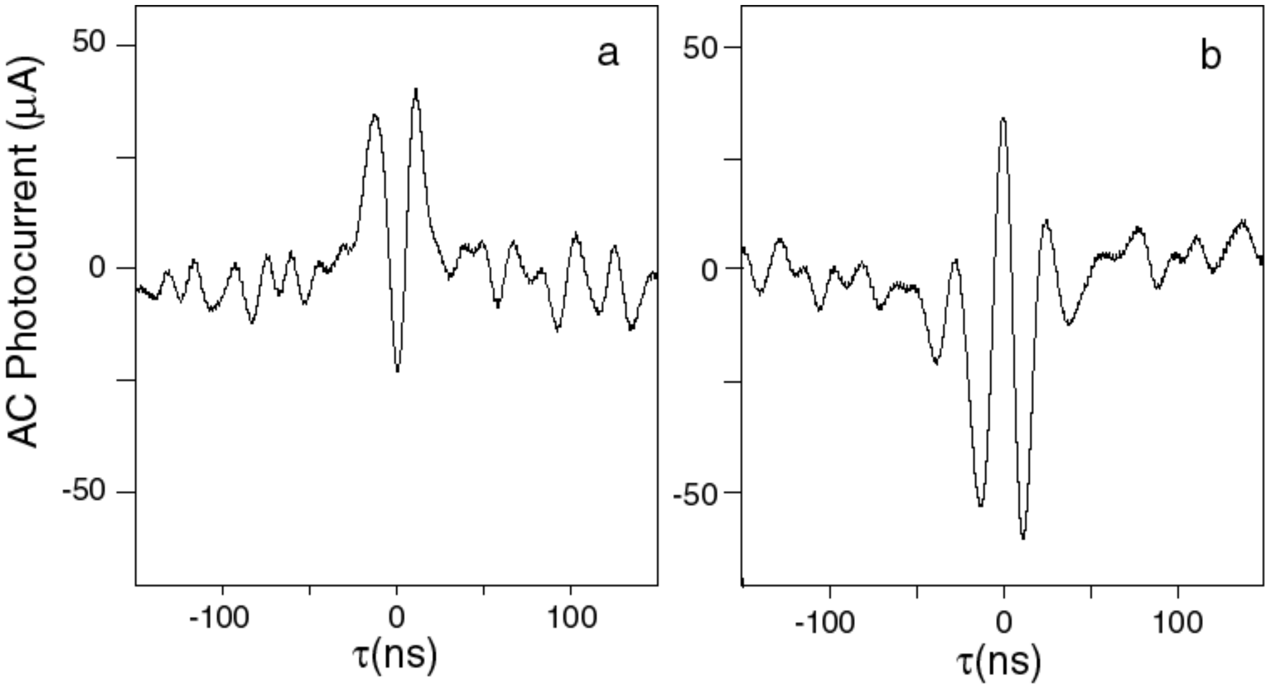
After 10,000 averages



After 30,000 averages

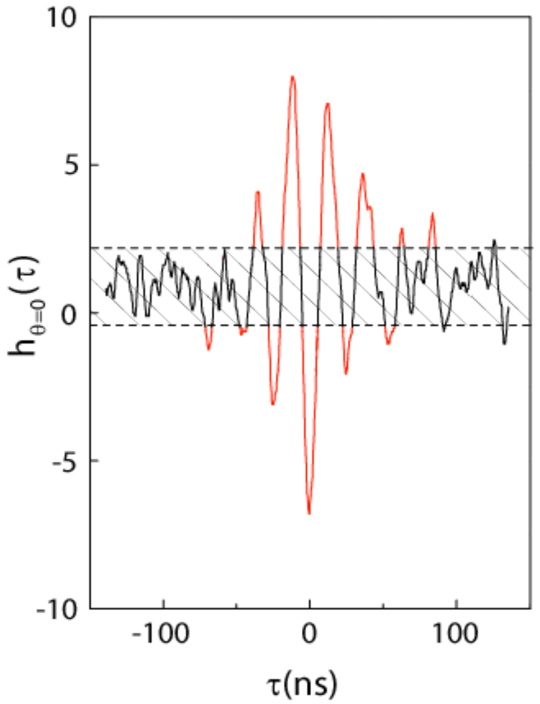


After 65,000 averages



Flip the phase of the Mach-Zehnder by 146°

Monte Carlo simulations for weak excitation:



Atomic beam N=11